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A note on the paper: Solving the job-shop scheduling problem optimally by dynamic programming

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Abstract

We point out a flaw in the arguments given in [6] for proving the correctness of the algorithm there proposed, by showing an example that contradicts some claims of that article. More importantly, we amend the flaw, providing a new and simpler proof of the correctness of the algorithm.

1. Introduction

The job-shop scheduling problem is given by n jobs and m machines. Each job has to visit all the machines following a specific order. The order in which each job has to be processed along the machines, and the time it requires in each machine are known. Machines can process only one job at a time and the same job can not be processed simultaneously in two different machines. The goal is to minimize the *makespan*, i.e.: the completion time of the complete set of jobs.

The job-shop scheduling problem is one of the most studied combinatorial optimization problems, but it remains a very challenging problem. Even 'simplified versions' of the job-shop scheduling problem are NP-Hard (see, for example, [4]).

In [6] an algorithm is proposed for solving the job-shop scheduling problem optimally using a dynamic programming strategy. This is, according to our knowledge, the first exact algorithm for the Job Shop problem which is not based on integer linear programming and branch and bound. Despite the correctness of the dynamic programming algorithm presented in [6], the proof of correctness given there is unfortunately flawed. The contribution of the present paper is threefold: first we show by means of a simple example where the flaw lies. Secondly, we present a correct and to some extent more intuitive proof. Thirdly, we establish the contribution of the aforementioned paper as correct and worth merit enabling subsequent research to improve on it.

For a brief review on the bibliography on job shop scheduling problems, we refer the reader to [6]. However, since the appearance of [6], many articles have been published, dealing with the job shop scheduling problem. To mention a few: in [1] neighbourhood strategies are considered. In [2] an enumerative parallelized algorithm is developed. Both [3, 11] propose modified genetic algorithms. Constrained and mixed integer programming are studied in [8], whereas a differential evolution algorithm is proposed in [10]. Finally, an heuristic method is developed for a variant of the job shop scheduling problem with tree-structured precedence constrains in [5]. All of these articles take [6] into account, but fortunately it is used there in ways that are not affected by the flaw that we notice and repair.

In Section 2 we introduce the notation and the main definitions for the problem formulation. In Section 3 we state the dynamic programming formulation for the job-shop scheduling problem, and the algorithm therefore obtained, whereas in Section 4 we briefly review the arguments given in [6] for proving the correctness of the algorithm, and present a counterexample that show that some of the claims of [6] do not hold. Finally, in Section 5 a new proof is given for the correctness of the algorithm. Taking this into account, it is very important to remark that any work based exclusively on the correctness of the algorithm would not be affected by the present paper. Only some proofs, and not the results, given in [6] should be revised.

2. Notation and Preliminaries

We denote $\mathcal{J} = \{j_1, \ldots, j_n\}$ the set of jobs and $\mathcal{M} = \{m_1, \ldots, m_m\}$ the set of machines. Each job consists of m operations that should be processed in a given order. We denote $\mathcal{O} = \{o_1, o_2, \ldots, o_n, \ldots, o_{nm}\}$ the set of all the operations. The first n operations correspond to the first operation of each job, whereas operations $o_{n+1}, \ldots, o_{2n}$ correspond to the second operations of each job, and so on. In this way, j_i is formed by operations: $\{o_{kn+i}\}_{k=0,\ldots,m-1}$. For each operation o, we denote $m(o)$ the machine in which o should be processed and $j(o)$ the job where o belongs. Observe that $j(o_i) = i \mod n$. Finally, we denote $p(o)$ the processing time of o in $m(o)$.

Following this notation an instance of the job shop scheduling problem is given by the numbers n and m of jobs and machines, and two vectors of length $n \times m$, containing $m(o)$ and $p(o)$ for each operation o.

Definition 2.1. A schedule is a function $\psi : \mathcal{O} \to \mathbb{N} \cup \{0\}$, where $\psi(o)$ gives the starting point of operation o. A schedule ψ is feasible if:

1. For all $o_k, o_l \in \mathcal{O}$ such that $j(o_k) = j(o_l)$ and $k < l$, $\psi(o_k) + p(o_k) \leq \psi(o_l)$.
2. For all $o_k, o_l \in \mathcal{O}$ such that $m(o_k) = m(o_l)$ we have that $\psi(o_k) + p(o_k) \leq \psi(o_l)$ or $\psi(o_l) + p(o_l) \leq \psi(o_k)$.

The goal of the job shop scheduling problem is to find a feasible schedule that minimizes

$$C_{\max}(\psi) = \max_o \{\psi(o) + p(o)\}.$$

Schedules are represented by Gantt charts, like the ones showed in Figure 1. Each row represents a machine, while the x axis is time. The bars represent operations, and colors are used to identify jobs.

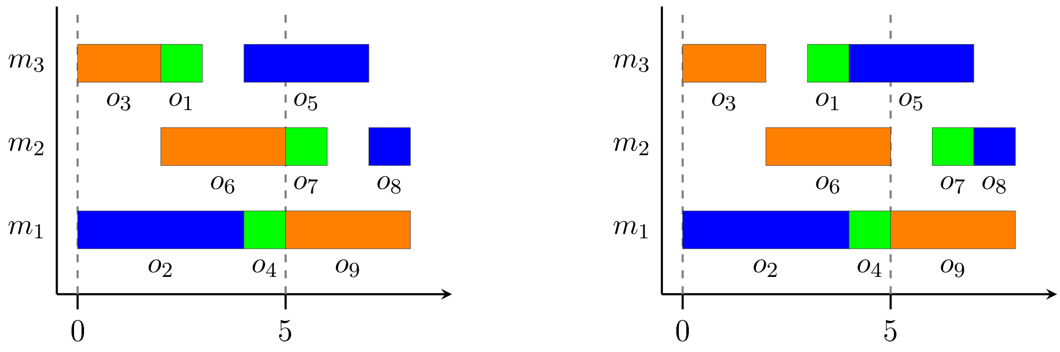


FIGURE 1. Active (left) and non-active (right) schedules.

Each schedule ψ can be associated to a sequence of operations by sorting the operations following some fixed criteria (for example: according to the starting time given by ψ). The following proposition is proved in [6] and establishes the criteria that we adopt for associating schedules to sequences of operations.

Proposition 2.1. *For every feasible solution for the job-shop scheduling problem there is one and only one sequence of operations defining the schedule such that the completion time of the operations along the sequence is non-decreasing and in which the order of the machines is increasing for two consecutive operations with equal completion time.*

Proposition 2.1 says that for each schedule we have one and only one sequence of operations. However, the converse is not true. Figure 1, for example, shows two possible schedules for the sequence: $o_3, o_1, o_2, o_4, o_6, o_7, o_4, o_9, o_8$. It is clear that any sequence admits an infinite number of schedules, since when all the operations in a tail of the sequence are moved to the right the same amount of time units, the relative order between them is not altered.

In order to identify sequences and schedules, we introduce, following for example [9], the notion of *active* schedules.

Definition 2.2. A feasible schedule ψ is *active* if the action of moving any operation one unit of time to the left makes it unfeasible.

In Figure 1, the left schedule is active, whereas the right one is not, since operations o_1 and o_7 have been moved unnecessarily to the right. Non-active schedules are also called *idle* schedules, since machines are idle even though jobs are available to process.

According to the notion of active schedule, given a sequence of operations, we will associate to it the schedule where the starting time for each operation is fixed as soon as possible, as long as it satisfies the restrictions with respect to the previous operations. Such a procedure guarantees that only active schedules are produced.

Remark 2.1. Given a sequence ς there is only one feasible active schedule ψ_ς associated to it. On the other hand, given the schedule ψ_ς, there is one and only one sequence ς' associated to it, according with Proposition 2.1. However, it is important to observe that ς' is not necessarily equal to ς. Consider, for the instance of Figure 1, the sequence $\varsigma = o_2, o_3, o_1, o_4, o_6, o_5, o_7, o_9, o_8$. ψ_ς is the schedule at the left of Figure 1, however ς is not ordered according to Proposition 2.1. The ordered sequence given by ψ_ς is $\varsigma' = \{o_3, o_1, o_2, o_6, o_4, o_7, o_5, o_8, o_9\}$.

We say that a sequence ς is *ordered*, if it is ordered according to the criteria established in Proposition 2.1. In other words: if $\varsigma' = \varsigma$. Otherwise, we say that ς is unordered. Furthermore, we denote $S(\varsigma)$ the subset of $\mathcal{O}$ containing all the operations that appear in the sequence ς. We say that ς is partial when $S(\varsigma) \neq \mathcal{O}$. On the other hand, ς is complete if $S(\varsigma) = \mathcal{O}$. In order to apply a dynamic programming strategy, we will build partial ordered sequences adding one operation at a time. The following definition states the basic notation for the dynamic programming formulation.

Definition 2.3. Given a sequence ς we define:

1. $\varepsilon(\varsigma) \subseteq \mathcal{O} \setminus S(\varsigma)$ is the set of operations o that can be added to ς such that all the operations in $j(o)$ that have to be scheduled before o belong to $S(\varsigma)$. Observe that $\varepsilon(\varsigma)$ depends only on $S(\varsigma)$, and not on the particular permutation ς.
2. We denote $\varsigma + o$ the sequence obtained by adding o at the end of ς. We say that $\varsigma + o$ is an expansion of ς. Observe that $\varsigma + o$ can be unordered, even if ς is ordered. We also denote $\psi(\varsigma, o)$ the starting time of o in $\varsigma + o$.
3. $\eta(\varsigma) \subseteq \varepsilon(\varsigma)$ is the set of operations such that $\varsigma + o$ is ordered. Observe that $\eta(\varsigma)$ depends on the sequence ς, and not only on the set of operations $S(\varsigma)$.
4. We say that $\varsigma_{\mathcal{O}}$ is a completion of ς if $S(\varsigma_{\mathcal{O}}) = \mathcal{O}$ and $\varsigma_{\mathcal{O}}$ is obtained from ς by sequentially adding one operation at a time.
5. For any (partial) sequence ς, $C_{\max}(\varsigma)$ stands for the completion time of ς.
6. Given $S \subseteq \mathcal{O}$, we denote $\Xi(S)$ the set of all ordered sequences ς such that $S(\varsigma) = S$.
7. For any sequence ς, we denote $\varsigma[i]$ the i-th operation of sequence ς.
8. For any sequence ς we denote $\Lambda(\varsigma)$ the last operation of ς.

3. The algorithm

In the seminal work of Held and Karp [7] several problems, including a simplified scheduling problem, are formulated as *sequencing* problems, and a dynamic programming approach is applied to them. Sadly, the strategy presented in [7] cannot be used for solving the job shop scheduling problem. The main difficulty for doing so is that the optimality principle does not hold for the job shop scheduling using the *natural* functional $C_{\max}$. Consequently, some technical work should be done in order to find a proper formulation for the application of dynamic programming. However, it is important to remark that we will not obtain a functional equation, as in classical dynamic programming, but a recursive strategy that will allow the progressive construction of the optimal solution.

Such a formulation for the job shop scheduling problem, and the exact algorithm that is derived from it are the main contributions of [6]. The complexity of the algorithm is exponential, but, more importantly, it is exponentially better than brute force.

Following a dynamic programming strategy, the algorithm proceeds in $n \times m$ stages. In stage i only ordered sequences of exactly i operations are considered. Some sequences are compared according to a criterion that is specified below, and that states a *domination* relationship between some sequences. When a sequence is *dominated* by another, it is discarded. For the sequences that are not discarded, all the possible ordered expansions are generated, obtaining sequences with $i + 1$ operations. At stage $n \times m$ an optimal solution is found.

In order to compare partial sequences, we define an *aptitude* value for a sequence ς and every operation $o \in \varepsilon(\varsigma)$:

$$\alpha(\varsigma, o) = \begin{cases} \psi(\varsigma, o) + p(o) & \text{if } o \in \eta(\varsigma), \\ C_{\max}(\varsigma) + p(o) & \text{otherwise.} \end{cases}$$

Observe that α is a lower bound for the completion time of o in any ordered completion of ς: if $o \in \eta(\varsigma)$, it can be added immediately, with completion time $\alpha(\varsigma, o)$, but it can also be added in a further step, with completion time greater that $\alpha(\varsigma, o)$; on the other hand, if $o \notin \eta(\varsigma)$ another operation o' has to be added before o in $m(o)$, with completion time at least $C_{\max}(\varsigma)$, and consequently when o is added its completion time is greater than $C_{\max}(\varsigma) + p(o)$.

We use α to compare partial solutions. It is noteworthy than only sequences involving the same operations can be compared. We denote $\vec{\alpha}(\varsigma)$ a vector containing the values of $\alpha(\varsigma, o)$, for every $o \in \varepsilon(\varsigma)$, ordered by job. Given two sequences ς^1 and ς^2 such that $S(\varsigma^1) = S(\varsigma^2)$, we say that $\vec{\alpha}(\varsigma^1) \lessdot \vec{\alpha}(\varsigma^2)$ if $\alpha(\varsigma^1, o) \leq \alpha(\alpha^2, o)$ for all o.

The following proposition is proved in [6], and it is the key of the proposed algorithm.

Proposition 3.1. *Let ς^1 and ς^2 be partial sequences in $\Xi(S)$, such that $\vec{\alpha}(\varsigma^2) \lessdot \vec{\alpha}(\varsigma^1)$ Then, every operation $o \in \mathcal{O} \setminus S$ of an* ordered *completion $\varsigma^1_{\mathcal{O}}$ of ς^1 can be scheduled at the same time in the schedule of ς^2. This leads to a feasible, though possibly non-active, complete schedule with makespan $C_{\max}(\varsigma^1_{\mathcal{O}})$.*

Remark 3.1. It is important to notice that the completion of ς^2 with the operations in $\mathcal{O} \setminus S$ in the order that they are scheduled in ς^1 can produce an *unordered* sequence.

Since the schedule obtained by completing ς^2 can be non-active, some operations can be moved to the left producing an active schedule ψ^2 with makespan lesser or equal than $C_{\max}(\varsigma^1_{\mathcal{O}})$. According to this result, we say that if $\vec{\alpha}(\varsigma^2) \lessdot \vec{\alpha}(\varsigma^1)$, ς^2 *dominates* ς^1.

It is possible to find sequences ς^1 and ς^2 such that $\alpha(\varsigma^1, o) = \alpha(\varsigma^2, o)$ for every o. In such cases some rule should be adopted in order to decide whether ς^1 dominates ς^2 or viceversa. It doesn't matter what rule is used, as long as the same criteria is applied to all the cases, for example take the lowest operation number of the first difference in the sequences.

Corollary 3.1. *Let ς^1 be dominated by ς^2. Then, Proposition 3.1 implies that for any ordered completion $\varsigma^1_{\mathcal{O}}$ of ς^1, there is an ordered complete sequence $\varsigma^2_{\mathcal{O}}$ with equal or lower makespan. However Remark 3.1 indicates that such sequence is not necessarily obtained by iteratively expanding ς^2.*

Corollary 3.1 contains both the core of the algorithm, and its main subtleties. As we commented earlier, dominated sequences are dropped. This seems to be allowed by the corollary: we drop ς^1 because we know that for any completion of ς^1 another solution with equal or lower makespan can be produced. However, it is possible that such a better solution does not come directly from ς^2, but from another partial sequence ς^3 that is not comparable to ς^1 at stage $|S|$. Moreover, it is theoretically possible that the algorithm never generates ς^3, if some partial sequence of it is dropped at a previous stage. Taking this into account, it is not obvious that an optimal solution should be produced. It is clear that if a certain instance of the problem admits only one optimal solution, it will be never discarded. But if there are two or more optimal solutions, it would be possible that they dominate each other at different stages making the algorithm drop all of them. Fortunately, such a situation is not really possible, as we prove in Section 5.

Aside from the domination criterion, that leads us to the dynamic programming formulation, [6] states a *state reduction* procedure that allows to drop some additional partial sequences, reducing the number of sequences considered by the algorithm and, therefore, its practical performance. This procedure is not affected by the flaws of the proofs given in [6] and it is consequently omitted here.

We conclude this section stating a simplified scheme of the algorithm:

Require: An instance of the job shop scheduling problem
Ensure: An optimal solution for the instance.
  for all $o \in \varepsilon(\emptyset)$ **do**
    Define the set $\mathcal{F}(\{\varsigma\}) = \{\varsigma\}$ with $\varsigma = (o)$.
  for $i = 1$ **to** $n \times m$ **do**
    for all $S \subset \mathcal{O} : \ |S| = i$ **do**
      for all $\varsigma \in \mathcal{F}(S)$ **do**
        for all $o \in \eta(\varsigma)$ **do**
          $\varsigma' = \varsigma + o$.
          if ς' is not dominated by any sequence $\varsigma^2 \in \mathcal{F}(S \cup \{o\})$ **then**
            for all $\varsigma^2 \in \mathcal{F}(S \cup \{o\})$ **do**
              if ς' dominates ς^2 **then**
                remove ς^2 from $\mathcal{F}(S \cup \{o\})$
              add ς' to $\mathcal{F}(S \cup \{o\})$
  return the sequence $\varsigma \in \mathcal{F}(\mathcal{O})$ with minimum $C_{\max}$.

4. The original proof

The formulation of the job-shop scheduling problem as a sequencing problem is developed in [6] along with the dynamic programming algorithm. In order to prove the correctness of the algorithm, many new notions are introduced, and several preparatory results are proven. Unhappily, problems have been found on some of these preliminary steps. For the sake of brevity, we only comment here a key point that makes the main proof to fail, and present a counterexample to show this.

We have already defined the set $\Xi(S)$ containing all the ordered sequences using the operations in S. In [6] two subsets of $\Xi(S)$ are defined. $\widehat{\Xi}(S)$ is formed by all the sequences in $\Xi(S)$ that are not dominated by any other sequence in $\Xi(S)$, and $\overset{\triangle}{\Xi}(S)$ is the set of all the sequences ς in $\widehat{\Xi}(S)$ such that all the subsequences of ς are in $\widehat{\Xi}(S')$ for the corresponding set S'.

Proposition 3 in [6] states that the set $\overset{\triangle}{\Xi}(S)$ is never empty. Based on this result, Proposition 4 concludes that the sets $\mathcal{F}(S)$ generated by the algorithm are exactly $\overset{\triangle}{\Xi}(S)$. The following example shows both these assertions to be false, invalidating the line of argumentation of [6]. Fortunately, as it is shown in the next section, these problems are not essential, and can be avoided following a slightly different path.

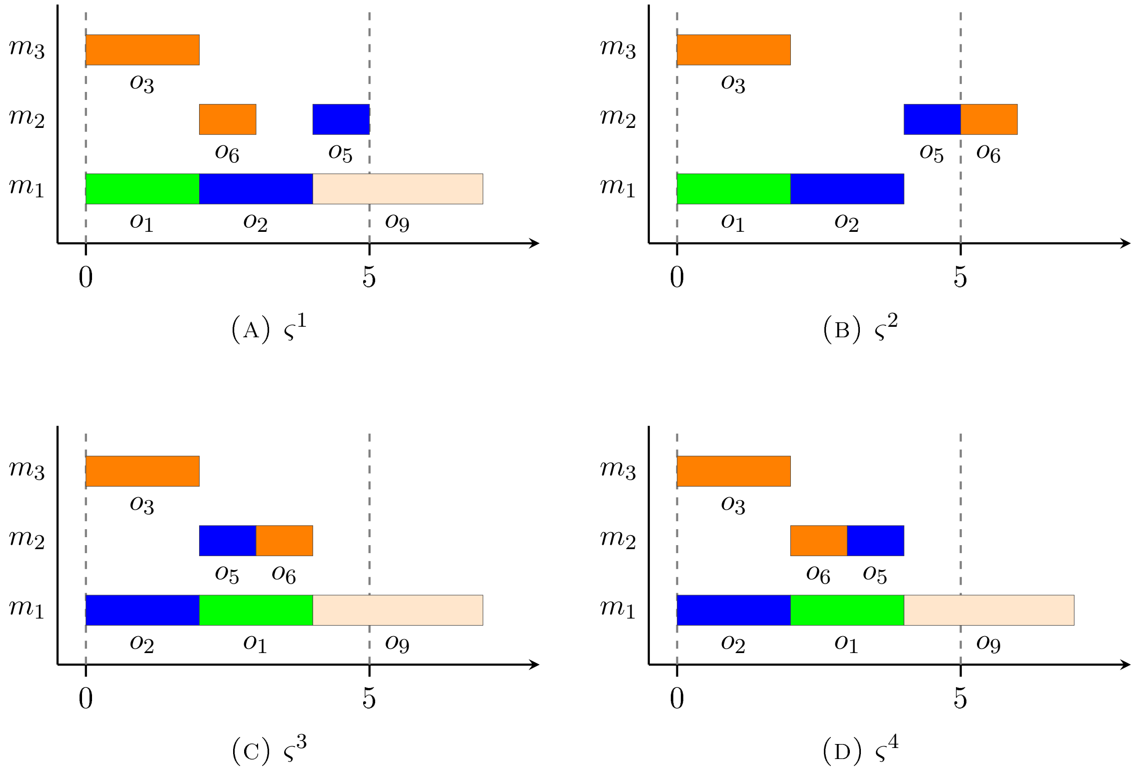
Consider the instance given by:

Operations	o_1	o_2	o_3	o_4	o_5	o_6	o_7	o_8	o_9
$p(o)$	2	2	2	4	1	1	1	3	3
$m(o)$	1	1	3	3	2	2	2	3	1

For this instance, consider the set $S = \{o_1, o_2, o_3, o_5, o_6\}$. It is easy to verify that $\Xi(S)$ is given by the four sequences:

$$\varsigma^1 = (o_1, o_3, o_6, o_2, o_5), \quad \varsigma^2 = (o_1, o_3, o_2, o_5, o_6),$$
$$\varsigma^3 = (o_2, o_3, o_5, o_1, o_6), \quad \varsigma^4 = (o_2, o_3, o_6, o_1, o_5),$$

represented by the following schedules:



(A) ς^1 (B) ς^2 (C) ς^3 (D) ς^4

FIGURE 2. Sequences in the set $\Xi(S)$

For these sequences we have:

$$\begin{array}{llll}
\alpha(\varsigma^1, o_4) = 6 & \alpha(\varsigma^2, o_4) = 6 & \alpha(\varsigma^3, o_4) = 8 & \alpha(\varsigma^4, o_4) = 8 \\
\alpha(\varsigma^1, o_8) = 8 & \alpha(\varsigma^2, o_8) = 8 & \alpha(\varsigma^3, o_8) = 6 & \alpha(\varsigma^4, o_8) = 7 \\
\alpha(\varsigma^1, o_9) = 7 & \alpha(\varsigma_2, o_9) = 9 & \alpha(\varsigma^3, o_9) = 7 & \alpha(\varsigma^4, o_9) = 7
\end{array}$$

So we conclude that $\varsigma^1 \lessdot \varsigma^2$ and $\varsigma^3 \lessdot \varsigma^4$. It is easy to check that the subsequences of ς^1 and ς^3 are in the corresponding $\widehat{\Xi}(S')$, so we conclude that:

$$\overset{\triangle}{\Xi}(S) = \{\varsigma^1, \varsigma^3\}.$$

Now, consider the set $S \cup \{o_9\}$. The algorithm could form sequences with set $S \cup \{o_9\}$ expanding, with the corresponding operation, not-dominated sequences with different sets of cardinal $|S|$. Particularly, with set: S, $(S \cup \{o_9\}) \setminus \{o_5\}$, $(S \cup \{o_9\}) \setminus \{o_6\}$ and $(S \cup \{o_9\}) \setminus \{o_1\}$. However, it is easy to see that the algorithm would only expand S, since the other alternatives are unfeasible or dominated at previous stages. Therefore, the algorithm would generate only the sequences $\varsigma^1 + o_9$ and $\varsigma^3 + o_9$. Observe that the expansions $\varsigma^1 + o_9$, $\varsigma^3 + o_9$ and $\varsigma^4 + o_9$ are shown in Figure 2.

For these sequences we have:

$$\begin{array}{cc} \alpha(\varsigma^1 + o_9, o_4) = 11 & \alpha(\varsigma^3 + o_9, o_4) = 8 \\ \alpha(\varsigma^1 + o_9, o_8) = 8 & \alpha(\varsigma^3 + o_9, o_8) = 10, \end{array}$$

which means that none of these sequences will be dropped. However, the sequence $\varsigma^4 + o_9$ is in $\Xi(S \cup \{o_9\})$, and we have:

$$\begin{array}{c} \alpha(\varsigma^4 + o_9, o_4) = 8 \\ \alpha(\varsigma_4 + o_9, o_8) = 7. \end{array}$$

This means that $\varsigma^4 + o_9$ dominates both $\varsigma^1 + o_9$ and $\varsigma^3 + 9$. Two main conclusions can be derived from this fact. The first one is that $\overset{\triangle}{\Xi}(S \cup \{o_9\})$ is empty, which contradicts Proposition 3 in [6]. The second is that the sets $\mathcal{F}(S)$ built by the algorithm are not the sets $\overset{\triangle}{\Xi}(S)$: since ς^4 would not be expanded to $\varsigma^4 + o_9$, this last sequence would not be available for comparison, and $\varsigma^1 + o_9$ and $\varsigma^3 + o_9$ would never be dropped. Therefore, we have that $\mathcal{F}(S \cup \{o_9\}) = \{\varsigma^1 + o_9, \varsigma^3 + o_9\}$, even when $\overset{\triangle}{\Xi}(S \cup \{o_9\}) = \emptyset$.

As we commented above, this example invalidates the course of action taken in [6]. However, the central ideas exposed there are still useful for proving the correctness of the algorithm, as proved in the next section.

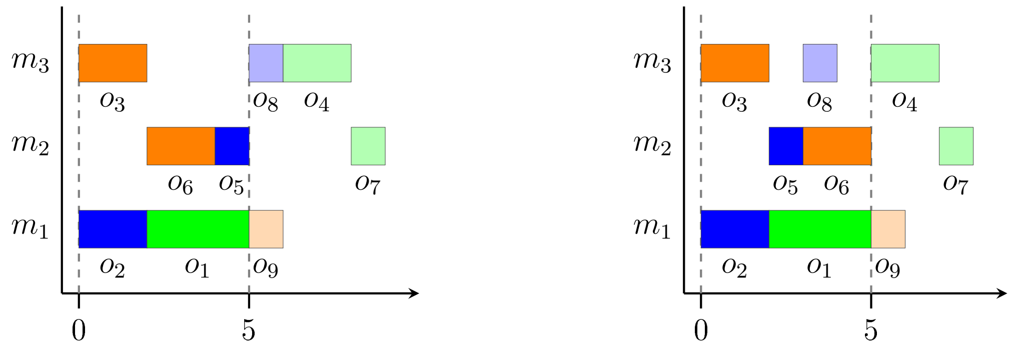
5. The new proof

Even though the proof provided in [6] is not correct, the algorithm does indeed provide an optimal solution. In this section we provide a new proof for the correctness of this algorithm.

As we have seen above it is possible that a partial solution ς^1, in particular a partial solution of an optimal solution, can be dominated by another partial solution ς^2 which does not have an ordered completion which produces at least the same makespan as the best completion of the dominated solution. We can show such a solution exist by adding any completion of the dominated solution ς^1 to the dominating partial solution ς^2. When this schedule is converted to a non-idle schedule due to the domination criteria the makespan is at least the same or better. However, the ordered sequence of such a solution is possibly not a completion of the dominating solution ς^2.

To prove that an optimal solution is found we show that not all optimal solutions can be dominated this way and an optimal solution must be found by the Dynamic Programming algorithm. To show this we need to establish a few extra properties of the state space of the Dynamic Programming algorithm.

Let ς^1_O be an complete solution that is not found by the Dynamic Programming algorithm. Than there must be a partial solution ς^1_S of ς^1_O that is dominated by another partial solution ς^2_S with the same set of operations S. When ς^1_S is dominated by ς^2_S we can distinguish two cases for any completion ς^1_O of ς^1_S. Let ς^2_O be the ordered sequence of the schedule created by adding all operations of the completion ς^1_O of ς^1_S to the dominating solution ς^2_S. We call the solution ς^2_O *welded* from ς^2_S and the completion to ς^1_O of ς^1_S. Now we can distinguish two cases for ς^2_O



(A) Schedule of $o_2o_3o_6o_1o_5$ with completion $o_9o_8o_4o_7$

(B) Schedule of $o_2o_3o_5o_1o_6$ where the completion of fig. 3a is welded

FIGURE 3. Indirect domination where operation o_8 of a completion is scheduled before the last operation of the dominating sequence

I. The welded sequence $\varsigma^2_{\mathcal{O}}$ starts with the sequence ς^2_S. This implies that the operations of the completion from ς^1_S to $\varsigma^1_{\mathcal{O}}$ can be added after ς^2_S in an ordered way: otherwise, an operation o should be inserted *before* the last operation in ς^2_S, and the first $|S|$ operations of $\varsigma^2_{\mathcal{O}}$ would not be equal to ς^2_S. We call this *direct* domination. Note that the order of the operations in the completion may differ.

II. The welded sequence $\varsigma^2_{\mathcal{O}}$ does not start with the sequence represented by the partial solution ς^2_S. This implies that at least one operation $o \in \mathcal{O} \setminus S$ in schedule of $\varsigma^2_{\mathcal{O}}$ is advanced such that this operation occurs in the ordered sequence before the last operation $\Lambda(\varsigma^2_S)$ of the sequence represented by ς^2_S. This implies that $\alpha(\varsigma^2_S, o) = C_{\max}(\varsigma^2_S) + p(o)$ as otherwise the expansion of o could be done in an ordered way. In this case, solution $\varsigma^2_{\mathcal{O}}$ cannot be produced by successive expansions of ς^2_S, but by expanding another solution ς^3, that is not comparable to ς^1_S at stage $|S|$. We call this *indirect* domination.

Figure 3 shows an example of two partial solutions where the partial solution $o_2o_3o_6o_1o_5$ in fig. 3a is dominated by the partial solution $o_2o_3o_5o_1o_6$ in fig. 3b. Also a completion is shown where the domination is indirect as can be seen in fig. 3a where operation o_8 should be in the sequence before operations o_1 and o_6. The partial solution $o_2o_3o_5o_8o_1$ leading to the complete solution in fig. 3b would be the partial solution belonging to the same stage as the two partial solutions depicted.

When we have indirect domination (Case II) we can deduce some special properties.

Proposition 5.1. *If we have indirect domination between ς^1 and ς^2 as described in Case II, there is at least an operation $o \in \mathcal{O} \setminus S$ that is scheduled in the welded solution $\varsigma^2_{\mathcal{O}}$ such that o is finished in $\varsigma^2_{\mathcal{O}}$ before it is scheduled to start in $\varsigma^1_{\mathcal{O}}$.*

Proof. As we have indirect domination there is at least one operation o that is scheduled in $\varsigma^2_{\mathcal{O}}$ before $\Lambda(\varsigma^2)$. As operation o could not be scheduled as expansion of ς^2_S leading to an ordered schedule we have the following

$$\psi(\varsigma^1_{\mathcal{O}}, o) + p(o) \geq \alpha(\varsigma^1_S, o) \geq \alpha(\varsigma^2_S, o) = C_{\max}(\varsigma^2_S) + p(o).$$

From this we can conclude that

$$\psi(\varsigma^1_{\mathcal{O}}, o) \geq C_{\max}(\varsigma^2_S) = \mathcal{O}(\varsigma^2_{\mathcal{O}}, \Lambda(\varsigma^2)) + p(\Lambda(\varsigma^2)) \geq \psi(\varsigma^2_{\mathcal{O}}, o) + p(o).$$

□

Corollary 5.1. *Operation o of Proposition 5.1 can be scheduled twice in $\varsigma^2_{\mathcal{O}}$ with a makespan equal or lower as that of $\varsigma^1_{\mathcal{O}}$.*

Proof. On one hand, operation o of Proposition 5.1 can be scheduled after $C_{\max}(\varsigma^2_S)$. On the other hand, it can be scheduled in the ordered sequence such that is finished before $C_{\max}(\varsigma^2_S)$. Therefore operation o can be scheduled twice consecutively in $\varsigma^2_{\mathcal{O}}$. □

We have seen that all operations of a completion of a dominated solution can be scheduled at the same time or earlier in the schedule of a dominating solution. We can also deduce another important property of domination, which considers not the operations individually but the location within the sequence. For this we denote with $\varsigma[i]$ the i-th operation of the sequence ς and we denote with $C_o(\varsigma)$ the finish time of operation o in solution ς.

Proposition 5.2. *Let partial solution ς^1_S of solution $\varsigma^1_{\mathcal{O}}$ be dominated in stage $i = |S|$ by solution ς^2_S. Let $\varsigma^2_{\mathcal{O}}$ be the solution welded from the completion of ς^1_S to $\varsigma^1_{\mathcal{O}}$ and ς^2_S. Then we have that for any $j > i = |S|$ that $C_{\varsigma^2_{\mathcal{O}}[j]}(\varsigma^2_{\mathcal{O}}) \leq C_{\varsigma^1_{\mathcal{O}}[j]}(\varsigma^1_{\mathcal{O}})$.*

Proof. When the completion from $\varsigma^1_{\mathcal{O}}$ to ς^1_S is scheduled (possibly idle) at the times of $\varsigma^1_{\mathcal{O}}$ after ς^2_S the proposition trivially holds. When this schedule is converted to a non-idle schedule operations are only moved backward in time. If this conversion is done in unit steps at the time it can be easily seen that the condition holds after each step. When an operation is moved backward by 1 without changing the order of operations the proposition naturally holds. When two operations must be switched to keep the ordering they have the same finish time just before the second operation is moved backward so the order of the operations can be changed without changing any finish time at any index. So at each index $j > i$ the finish time can only decrease. □

When a partial solution ς^1_S is dominated by ς^2_S we have the guarantee that for each completion $\varsigma^1_{\mathcal{O}}$ another (welded) solution $\varsigma^2_{\mathcal{O}}$ with equal or lower makespan exists, however, we do not yet have the guarantee that such a solution is found. It is possible, with indirect domination, that a dominating solution ς^2_S did not have an ordered completion with equal or lower makespan as $\varsigma^1_{\mathcal{O}}$. To show that we cannot dominate all optimal solutions we need the following proposition.

Proposition 5.3. *Let be $\varsigma^1_{\mathcal{O}}$ a solution and let its partial solution ς^1_S be dominated indirectly by ς^2_S in stage $i = |S|$. Let $\varsigma^2_{\mathcal{O}}$ be the solution welded from ς^2_S and the expansion of ς^1_S to $\varsigma^1_{\mathcal{O}}$. Now let for $k \geq 2$ $\varsigma^k_{S_k}$ be a partial solution of $\varsigma^k_{\mathcal{O}}$ that is directly or indirectly dominated by another partial solution $\varsigma^{k+1}_{S_k}$. Let $\varsigma^{k+1}_{\mathcal{O}}$ be the solution welded from $\varsigma^{k+1}_{S_k}$ and the expansion from $\varsigma^k_{S_k}$ to $\varsigma^k_{\mathcal{O}}$. When all dominations occur at or before stage i, thus $|S_k| \leq i$, we have $\varsigma^k_{\mathcal{O}} \neq \varsigma^1_{\mathcal{O}}$ for all welded solutions with $k \geq 2$.*

Proof. Since ς^2_S dominates ς^1_S indirectly there exist an operation $o \in \mathcal{O} \setminus S$ that is scheduled in $\varsigma^2_{\mathcal{O}}$ before the last operation $\Lambda(\varsigma^2_S)$. This operation o is scheduled in $\varsigma^1_{\mathcal{O}}$ such that $\psi(\varsigma^1_{\mathcal{O}}, o) \geq C_{\Lambda(\varsigma^2_S)}(\varsigma^2_S)$. First we conclude that the index of $\Lambda(\varsigma^2_S)$ is at least $i+1$ in $\varsigma^2_{\mathcal{O}}$. Using Proposition 5.2 and the fact that all dominations occur before stage $i+1$ we conclude that for all solutions $\varsigma^k_{\mathcal{O}}$ with $k \geq 2$ we have for operation $\varsigma^k_{\mathcal{O}}[i+1]$ that $C_{\varsigma^k_{\mathcal{O}}[i+1]}(\varsigma^k_{\mathcal{O}}) \leq C_{\Lambda(\varsigma^2_S)}(\varsigma^2_{\mathcal{O}})$. When $C_o(\varsigma^k_{\mathcal{O}}) \leq C_{\Lambda(\varsigma^2_S)}(\varsigma^2_{\mathcal{O}})$ we can conclude that $C_o(\varsigma^{k+1}_{\mathcal{O}}) \leq C_{\Lambda(\varsigma^2_S)}(\varsigma^2_{\mathcal{O}})$. When $o \notin S_k$ this follows directly from the domination and when $o \in S_k$ this follows from the fact that we have an ordered sequence, $|S_k| \leq i$ and that $C_{\varsigma_{\mathcal{O}}^{k}[i+1]}(\varsigma_{\mathcal{O}}^{k}) \leq C_{\Lambda(\varsigma_S^2)}(\varsigma_{\mathcal{O}}^{2})$. So in all solutions $\varsigma_{\mathcal{O}}^{k}$ with $k \geq 2$ we have that operation o finishes before it even starts in $\varsigma_{\mathcal{O}}^{1}$, and therefore $\varsigma_{\mathcal{O}}^{k} \neq \varsigma_{\mathcal{O}}^{1}$. □

Corollary 5.2. *When partial solution ς_S^1 of solution $\varsigma_{\mathcal{O}}^1$ is dominated in the DP algorithm before the last stage in stage $i = |S|$ ($i < |\mathcal{O}|$) there exists a partial solution in stage $i+1$ with a completion with a makespan not higher as $\varsigma_{\mathcal{O}}^1$.*

Proof. We have two cases:

(a) If ς_S^2 dominates $\varsigma_{\mathcal{O}}^1$ directly, the expansion of ς_S^2 with the first operation of the completion from ς_S^1 to $\varsigma_{\mathcal{O}}^1$ is ordered, and then the algorithm will perform the expansion, generating a partial sequence of $\varsigma_{\mathcal{O}}^2$ at stage $i+1$, which concludes the proof.

(b) If the domination is indirect, we consider now the sequence $\varsigma_{\mathcal{O}}^2$: if this sequence is generated, it is clear that, in particular, the subsequence of $\varsigma_{\mathcal{O}}^2$ containing the operations $\varsigma_{\mathcal{O}}^2[1], \ldots, \varsigma_{\mathcal{O}}^2[i+1]$ is built by the algorithm, and the result follows. On the other hand, if $\varsigma_{\mathcal{O}}^2$ is not generated, some partial sequence $\varsigma_{S_2}^2$ of $\varsigma_{\mathcal{O}}^2$ is dominated by some $\varsigma_{S_2}^3$. Iterating this process we find a chain of sequences $\varsigma_{\mathcal{O}}^k$ such that $\varsigma_{S_k}^{k+1}$ dominates $\varsigma_{S_k}^k$, as in Proposition 5.3. As all sequences $\varsigma_{\mathcal{O}}^k$ have a makespan not higher as $\varsigma_{\mathcal{O}}^1$ when any of the dominations in the chain occur in stage $i+1$ or higher the result follows. On the other hand when all dominations occur in stage i or lower there must exist a cycle in the chain of welded sequences $\varsigma_{\mathcal{O}}^k$ since there exists only a finite number of solutions. In order to prove that there is no cycle at all, we argue by contradiction: Let us assume a cycle in a chain of sequences as in Proposition 5.3 with all dominations in stage i or lower. Without loss of generality let $j \leq i$ be the largest stage where any domination occurs in this cycle. Any domination in this cycle at stage j is indirect as otherwise a partial solution of the dominating solution exists in stage $j+1$. Now observe such indirect domination, then Proposition 5.3 can be applied as all the dominations occur at stage j or lower. This directly leads to a contradiction with the existence of this cycle, and the results follows.

□

With these ingredients we can prove that the DP algorithm finds an optimal solution

Proposition 5.4. *The DP algorithm described in* [6] *finds an optimal solution for the job-shop scheduling problem.*

Proof. Suppose an optimal solution $\varsigma_{\mathcal{O}}^1$ is dominated, then there is a partial solution ς_S^1 of $\varsigma_{\mathcal{O}}^1$ that is dominated in stage $i = |S|$ by another partial solution ς_S^2. If $i < |\mathcal{O}|$ Corollary 5.2 provides a partial solution in stage $i+1$ with an optimal completion. Using this iteratively this provides an optimal solution in stage $|\mathcal{O}|$ where it can only be dominated directly by another optimal solution. So the DP algorithm described in [6] provides an optimal solution for the job-shop scheduling problem. □

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